

Plant Growth Model (PGM)

Technical Documentation

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Introduction

Increasing atmospheric carbon dioxide concentration affects plants in many ways. There are direct effects due to the elevated concentration of CO₂ and there are expected indirect effects that will be caused by the changes in temperature expected with global warming. In order to simulate the effects on plant water use and yield, a set of algorithms were added to WEAP that simulate the following processes.

1. Reduction in stomatal conductance caused by elevated CO₂.
2. Increase in radiation use efficiency caused by elevated CO₂ (fertilization effect).
3. Increase in leaf area caused by elevated CO₂.
4. Increases in evaporation and transpiration caused by elevated temperature.
5. Increase or decrease in temperature stress caused by elevated temperature.
6. Acceleration in the accumulation of degree day heat units which accelerates crop maturation.
7. Increase the length of the growing season caused by elevated temperature.
8. Reduction in stomatal conductance and radiation use efficiency caused by elevated vapor pressure deficit,

These processes are discussed in several publications (Kimball et al., 2002; Huntington, 2004; Neitsch, et al., 2005; Long et al., 2006; Ainsworth and Long, 2005; Hatfield et al., 2008; Kimball, 2010) and the reader is urged to read them for more details. At this time, there are no algorithms for the interactions between plants and nutrients in PGM. Published reports indicate that there are important interactions between nutrients, atmospheric CO₂ concentration and plant growth (Bloom, 2006; Bloom et al., 2010).

The algorithms implemented in the model were drawn from three main sources. The evapotranspiration calculations were largely extracted from the ASCE EWRI standardized reference crop ET calculations (ASCE EWRI, 2005). In the case of variables related to crops other than the standard crops described in ASCE EWRI (2005) the routines found in SWAT version 2005 were utilized (Neitsch, et al., 2005). Calculations of crop growth and yield were based on the routines described in the SWAT and APEX models (Neitsch, et al., 2005; Williams et al., 2008) with modifications for increase in leaf area index caused by elevated CO₂ concentration (Eckhardt, et al., 2002). Soil water balance calculations are similar to those found in the SWAT and APEX models (Neitsch, et al., 2005; Williams et al., 2008).

Potential Evapotranspiration

Introduction

In this model the alfalfa reference, as described in ASCE EWRI (2005) is used as the reference crop.

In the description of the evapotranspiration algorithm that follows, the source of each equation is provided. Equations were taken from the ASCE EWRI standardized reference evapotranspiration document (ASCE EWRI, 2005) and SWAT documentation (Neitsch, et al., 2005).

The model estimates potential evapotranspiration for each daily time step using the approach found in SWAT:

1. The potential evapotranspiration is initially estimated for the alfalfa reference crop (PETDAY) using the Penman-Monteith method.
2. The maximum plant evapotranspiration (EPMAX) is estimated using the Penman-Monteith method for specific crops such as annuals, and deciduous and non-deciduous perennial crops.
3. Evaporation from the crop canopy is calculated as a function of the size of the crop canopy and available moisture.
4. Potential bare soil evaporation is calculated as a function of canopy cover.
5. The sum of canopy evaporation, crop transpiration, and bare soil evaporation is compared to PETDAY. If the sum exceeds PETDAY then potential bare soil evaporation and maximum plant transpiration (EPMAX) are reduced, in that order.

Potential Evapotranspiration for the Alfalfa Reference Crop (PETDay)

$$PET_{Day} = \frac{DLT * RN + \rho * c_p * 86400 * VPD / AR}{HV * (DLT + GMA * [1 + CR / AR])}$$

Eq. 1

Where

PETDay: potential plant transpiration in mm d⁻¹ [Eq. 2:2.2.1 in SWAT 2005]

DLT: slope of the saturation vapor pressure curve in kPa °C⁻¹

RN: net radiation in MJ m⁻² d⁻¹

rho: air density in kg m⁻³

cp : specific heat of moist air at constant pressure in $\text{MJ kg}^{-1} \text{ } ^\circ\text{C}^{-1}$
 VPD : vapor pressure deficit in kPa
 AR : aerodynamic resistance for heat and vapor transfer in s m^{-1}
 HV : latent heat of vaporization in MJ kg^{-1}
 GMA : psychrometer constant in $\text{kPa } ^\circ\text{C}^{-1}$
 CR : canopy resistance for vapor transfer in s m^{-1}

To calculate potential evapotranspiration, the Penman-Monteith method must be solved for a reference crop. The model uses alfalfa at a height of 40 cm with a minimal leaf resistance of 100 s m^{-1} . The terms necessary to solve the Penman-Monteith equation for the alfalfa reference crop are as follows:

- a) The slope of saturation vapor pressure curve is calculated using the following equation:

$$DLT = \frac{4098 \exp\left(\frac{17.27 * T}{T + 237.3}\right)}{(T + 237.3)^2} \quad \text{Eq. 2}$$

Where

DLT : slope of saturation vapor pressure curve in $\text{kPa } ^\circ\text{C}^{-1}$ [Eq. 5 in ASCE EWRI]
 T : daily mean air temperature $([T_{\min} + T_{\max}] / 2)$ in $^\circ\text{C}$

- b) The net radiation for PET is calculated using the following equation:

$$RN_PET = ralb + rout \quad \text{Eq. 3}$$

Where

RN_PET : net radiation for PET in $\text{MJ m}^{-2} \text{ d}^{-1}$ [Eq. 15 in ASCE EWRI]
 $ralb$: net short-wave radiation for PET in $\text{MJ m}^{-2} \text{ d}^{-1}$
 $rout$: net outgoing long-wave radiation in $\text{MJ m}^{-2} \text{ d}^{-1}$

$$ralb = ra * (1.0 - 0.23) \quad \text{Eq. 4}$$

Where

$ralb$: [Eq. 16 in ASCE EWRI]
 ra : extraterrestrial radiation in ASCE EWRI or daily mean short-wave radiation in $\text{MJ m}^{-2} \text{ d}^{-1}$

$$rout = rbo * rto * 4.9E - 9 * \left[\frac{T_{K \max}^4 + T_{K \min}^4}{2} \right] \quad \text{Eq. 5}$$

Where

r_{out} : [Eq. 17 in ASCE EWRI]

$T_{K\ max}$: maximum absolute temperature during the 24-hour period [K]
(K=°C+273.16)

$T_{K\ min}$: minimum absolute temperature during the 24-hour period [K]
(K=°C+273.16)

$$r_{bo} = -(0.34 - 0.139 * \sqrt{ED}) \quad \text{Eq. 6}$$

Where

r_{bo} : net emissivity [Eq. 17 in ASCE EWRI]

There are two options for calculating the actual vapor pressure. One takes into consideration the min and max relative humidity, the second option determines the vapor pressure using dew point temperature. If the dew point temperature data are available, it is the preferred method (ASCE EWRI, 2005).

Min and Max Relative Humidity approach for determining vapor pressure

ED : Actual vapor pressure as average of max and min vapor pressure
in kPa

$$ED = \frac{svp_{min} * maxRH/100 + svp_{max} * minRH/100}{2} \quad \text{Eq.7}$$

Where

ED : [Eq. 11 in ASCE EWRI]

svp_{min} : minimum saturation vapor pressure using the ASCE EWRI
approach in kPa

svp_{max} : maximum saturation vapor pressure using the ASCE EWRI
approach in kPa

$maxRH$: maximum relative humidity in percent

$minRH$: minimum relative humidity in percent

$$svp_{min} = 0.6108 * e^{\frac{17.27 * MinTemp}{237.3 + MinTemp}} \quad \text{Eq.8}$$

$$svp_{max} = 0.6108 * e^{\frac{17.27 * MaxTemp}{237.3 + MaxTemp}} \quad \text{Eq.9}$$

$$SVP = \frac{svp_{min} + svp_{max}}{2} \quad \text{Eq. 10}$$

Where

svp_{min} : [Eq. 7 in ASCE EWRI]

svp_{max} : [Eq. 7 in ASCE EWRI]

SVP : saturation vapor pressure in kPa [Eq. 6 in ASCE EWRI]

$MinTemp$: minimum temperature in °C

$MaxTemp$: maximum temperature in °C

Dew Point Temperature approach for determining vapor pressure

ED: Actual vapor pressure using dew point temperature in kPa

$$ED = 0.6108 * e^{\frac{17.27 * Dew\ Point\ Temp}{237.3 + Dew\ Point\ Temp}} \quad \text{Eq.11}$$

Where

ED: [Eq. 8 in ASCE EWRI]

Dew Point Temp: dew point temperature in °C

The cloudiness function is estimated as follows:

$$r_{to} = 1.35 * \left(\frac{R_s}{RM_x} \right) - 0.35 \quad \text{Eq. 12}$$

Where

r_{to}: Cloudiness function [dimensionless] (limited to 0.05 ≤ *r_{to}* ≤ 1.0)

[Eq. 18 in ASCE EWRI]

R_s/R_{M_x}: relative solar radiation (limited to 0.3 ≤ *R_s/R_{so}* ≤ 1.0)

R_s: measured or calculated solar radiation for the day in MJ m⁻² d⁻¹

R_{M_x}: calculated clear-sky radiation in MJ m⁻² d⁻¹

The ratio *R_s/R_{M_x}* in Eq. 11 represents relative cloudiness and is limited to 0.3 < *R_s/R_{M_x}* ≤ 1.0 so that *r_{to}* has limits of 0.05 ≤ *r_{to}* ≤ 1.0.

To calculate the maximum possible radiation for the day, the solar declination, the relative distance of the earth from the sun, the sine and cosine of the site's latitude, and the corresponding Julian day have to be considered.

Solar declination:

$$sd = 0.409 * \text{Sin} \left(\frac{\text{JulianDay}}{58.09} - 1.39 \right) \quad \text{Eq. 13}$$

Where

sd: solar declination in radians [Eq. 24 in ASCE EWRI]

The eccentricity of the orbit is calculated as:

$$dd = 1 + 0.033 * \text{Cos} \left(\frac{\text{JulianDay}}{58.09} \right) \quad \text{Eq. 14}$$

Where

dd: inverse relative distance factor (squared) for the earth-sun [unitless]
[Eq. 23 in ASCE EWRI]

Sine and Cosine of the site's latitude:

$$\mathbf{latsin = Sin (Lat * 2\pi/360)} \quad \mathbf{Eq. 15}$$

$$\mathbf{latcos = Cos (Lat * 2\pi/360)} \quad \mathbf{Eq. 16}$$

The sunset hour angle, h, is given by:

$$\mathbf{h = ArcCos[-tan (lat) tan(sd)]} \quad \mathbf{Eq. 17}$$

$$\mathbf{ys = latsin * Sin (sd)} \quad \mathbf{Eq. 18}$$

$$\mathbf{yc = latcos * Cos (sd)} \quad \mathbf{Eq. 19}$$

Where

h: [Eq. 27 in ASCE EWRI]

Extraterrestrial radiation, ra , defined as short-wave solar radiation in the absence of an atmosphere. It is a well-behaved function of the time of the year and latitude. It is needed for calculating RMx , which is in turn used in calculating Rn . For daily (24-hour) periods, Ra can be estimated from the solar constant, the solar declination and the day of the year as follows:

$$\mathbf{ra = 37.586 * dd * (h * ys + yc * Sin (h))} \quad \mathbf{Eq. 20}$$

Where

ra: [Eq. 24 in ASCE EWRI]

When a dependable, locally calibrated procedure for determining RMx is not available, RMx , for purposes of calculating RN , can be computed as:

$$\mathbf{RMx = (0.75 + 2 * 10^{-5} Elev) ra} \quad \mathbf{Eq. 21}$$

Where

RMx: [Eq. 19 in ASCE EWRI]

Elev: station elevation above sea level in m

The net radiation for maximum plant evapotranspiration (ET) is calculated by the following equation:

$$\mathbf{RN_ET = ralb1 + rout} \quad \mathbf{Eq. 22}$$

Where:

RN_ET: net radiation for maximum plant ET in MJ m⁻² d⁻¹ [Eq. 42 in ASCE EWRI]

ralb1: net short-wave radiation for maximum plant ET in MJ m⁻² d⁻¹

$$\mathbf{ralb1 = ra * (1.0 - albdlay)} \quad \mathbf{Eq. 23}$$

Where:

ralb1: [Eq. 43 in ASCE EWRI]
albdlay: surface albedo for the day

To calculate the albedo for the day, the residue on soil surface for current day has to be determined.

$$\mathbf{SolCov = Max (0.8 * [PBio + Residue], 0.0)} \quad \mathbf{Eq. 24}$$

$$\mathbf{eaj = Exp (cej * [SolCov + 0.1])} \quad \mathbf{Eq. 25}$$

$$\mathbf{albdlay = 0.23 * (1.0 - eaj) + salb * eaj} \quad \mathbf{Eq. 26}$$

if the crop type is non-deciduous and completely covers the soil:

$$\mathbf{albdlay = 0.23} \quad \mathbf{Eq. 27}$$

Where:

SolCov: aboveground biomass and residue for current day in Tonnes/ha [SWAT 2005]
PBio: potential biomass production for current day in Tonnes/ha
Residue: crop residue on soil surface after harvest in Tonnes/ha
eaj: soil cover index [Eq. 1:1.2.16 in SWAT 2005]
cej: constant (-5×10^{-5})
salb: soil albedo for wet bare soil (0.08)
albdlay: [Eq. 1:1.2.15 in SWAT 2005]

c) The psychrometric constant is calculated by the following equation:

$$\mathbf{GMA = \frac{1.013E - 3 * PB}{0.622 * HV}} \quad \mathbf{Eq. 28}$$

Where:

GMA: [Eq. B.12 in ASCE EWRI]

$$\mathbf{PB = 101.3 (1.0 - 2.21E - 5 * ELEV)^{5.257}} \quad \mathbf{Eq. 29}$$

Assuming reference temperature of 293K – see ASCE EWRI Eq. 3

$$\mathbf{HV = 2.501 - 2.361E - 3 * TX} \quad \mathbf{Eq. 30}$$

PB: barometric pressure in kPa [Eq. 3 in ASCE EWRI]

HV: laten heat of vaporation in MJ/kg [Eq. B.7 in ASCE EWRI]

ELEV: elevation of the site in masl
TX: average daily air temperature in °C

d) The specific heat of moist air at constant pressure is as follow in MJ kg⁻¹ °C⁻¹:

$$cp = 1.013E - 3 \quad \text{Eq. 31}$$

e) The air density, rho, (kg/m³) is calculated by the following equation:

$$\rho = 3.486 * PB / Tkv \quad \text{Eq. 32}$$

Where:

rho: [Eq. B.10 in ASCE EWRI]

$$Tkv = \frac{TX + 273.16}{1.0 - 0.378 * \left(\frac{ED}{PB}\right)} \quad \text{Eq. 33}$$

Tkv: virtual temperature (°K) [Eq. B.11 in ASCE EWRI]

f) Vapor pressure deficit (kPa) is calculated by the following equation:

$$VPD = SVP - ED \quad \text{Eq. 34}$$

Where:

VPD: vapor pressure deficit in kPa [Eq. 1:2.3.5 in SWAT 2005]

SVP: saturation vapor pressure at mean air temperature in kPa

ED: vapor pressure at mean air temperature in kPa

g) The aerodynamic resistance is calculated by the following equation:

$$AR = \frac{114.0}{U2} \quad \text{Eq. 35}$$

Where

AR: aerodynamic resistance in s/m [Eq. 2:2.2.20 in SWAT 2005]

U2: mean daily wind speed at 2 m height in m s⁻¹

h) The canopy resistance is calculated by the following equation:

$$CR = \frac{49}{1.4 - 0.4 * (CO2 / 330)} \quad \text{Eq. 36}$$

Where:

CR: canopy resistance in s/m [Eq. 2:2.2.22 in SWAT 2005]

CO2: current atmospheric carbon dioxide concentration in ppm

Maximum Plant Evapotranspiration (EPMax)

To calculate the maximum plant evapotranspiration (*EPMax*) for a specific crop, the Penman-Monteith method is solved as follows:

$$EPMax = \frac{DLT * RN_ET + rho * cp * 86400 * VPD / ARMxET}{HV * (DLT + GMA * (1 + CRMxET / ARMxET))} \quad \text{Eq. 37}$$

Where

EPMax: maximum plant evapotranspiration for a specific crop in mm d⁻¹
[Eq. 2:2.2.1 in SWAT 2005]

DLT: slope of the saturation vapor pressure curve in kPa °C⁻¹

RN_ET: net radiation for maximum plant ET in MJ m⁻² d⁻¹ [Eq. 42 in ASCE EWRI]

rho: air density in kg m⁻³

cp: specific heat of moist air at constant pressure in MJ kg⁻¹ °C⁻¹

VPD: vapor pressure deficit in kPa

ARMxET: aerodynamic resistance for maximum plant ET in s m⁻¹

HV: latent heat of vaporization in MJ kg⁻¹

GMA: psychrometric constant in kPa °C⁻¹

CRMxET: Canopy resistance for maximum plant ET in s m⁻¹

To make sure maximum ET is not greater than potential ET (reference crop: Alfalfa)

$$EPMax = \text{Min} (EPMax, PETDay) \quad \text{Eq. 38}$$

Where:

EPMax: Maximum evapotranspiration for a specific crop in mm d⁻¹
[SWAT 2005 Code]

- a) The wind speed and height of wind speed measurement is calculated by the following equations based on the approach taken in SWAT.

If the crop height is less than 1.0 m (CPHT<1.0) in height, the wind speed is adjusted as follows:

$$UZZM_xET = U2 \quad \text{Eq. 39}$$

$$ZZM_xET = 200 \quad \text{Eq. 40}$$

$$ZOM = 0.123 * CHZ \quad \text{Eq. 41}$$

If the crop height is greater than 1.0 m and less than or equal to 2.5 m ($1.0 < CPHT \leq 2.5$) in height, the wind speed is adjusted as follows:

$$ZZM_xET = CPHT * 100 + 100 \quad \text{Eq. 42}$$

$$ZOM = 0.123 * CHZ \quad \text{Eq. 43}$$

$$UZZM_xET = U2 * \left(\frac{\text{Log}\left(\frac{ZZM_xET - D}{ZOM}\right)}{\text{Log}\left(\frac{200 - D}{ZOM}\right)} \right) \quad \text{Eq. 44}$$

If the crop height is greater than 2.5 m ($CPHT > 2.5$) in height, the wind speed is adjusted as follows:

$$ZZM_xET = CPHT * 100 + 100 \quad \text{Eq. 45}$$

$$UZZM_xET = U2 * (ZZM_xET/200)^{0.2} \quad \text{Eq. 46}$$

$$ZOM = 0.058 * CHZ^{1.19} \quad \text{Eq. 47}$$

Where:

$UZZM_xET$: wind speed ($m s^{-1}$) at height ZZ (cm) [Eq. B.14 in ASCE EWRI]

ZZM_xET : height at which wind is determined in cm [Eq. B.14 in ASCE EWRI]

$CPHT$: canopy height in m

ZOM : roughness length for momentum transfer in cm [Eq. B.14 in ASCE EWRI]

CHZ : canopy height in cm

b) The canopy height is calculated by the following equation. If crop height is less than 0.01 m, canopy height is as follows:

$$CHZ = 1.0 \quad \text{Eq. 48}$$

Otherwise

$$CHZ = CPHT * 100 \quad \text{Eq. 49}$$

Where

CHZ : [SWAT 2005 Code]

c) The roughness length for vapor transfer is calculated by the following equation.

$$ZOV = 0.1 * ZOM \quad \text{Eq. 50}$$

Where:

ZOV : roughness length for vapor transfer in cm [Eq. 2:2.2.6 in SWAT 2005]

The zero-plane displacement of wind profile is calculated by the following equation.

$$D = 0.667 * CHZ \quad \text{Eq. 51}$$

Where:

D : displacement height for plant type in cm [Eq. 2:2.2.7 in SWAT 2005]

d) The aerodynamic resistance for maximum plant ET is calculated by the following equation.

$$ARM_{xET} = \frac{\ln\left[\frac{(ZZM_{xET}-D)}{ZOM}\right] * \ln\left[\frac{ZZM_{xET}-D}{ZOV}\right]}{(0.41^{^2}) * UZZM_{xET}} \quad \text{Eq. 52}$$

Where:

ARM_{xET} : aerodynamic resistance for maximum plant ET in $s\ m^{-1}$
[Eq. 2:2.2.3 in SWAT 2005]

e) The stomatal conductivity is adjusted for high vapor pressure according to Figure 1 and it is calculated by the following equations.

$$\begin{aligned} F_{vpdMxET} &= \text{Max}(0.1, 1.0 - bx * XX) & \text{if } XX > 0 \\ F_{vpdMxET} &= 1.0 & \text{if } XX < 0 \end{aligned} \quad \text{Eq. 53}$$

$$gsi_{adj} = gsi * F_{vpdMxET} \quad \text{Eq. 54}$$

Where:

$$XX: VPD - vpth \quad \text{Eq. 55}$$

$$bx = (1 - vpd2) / (vpdabth - vpth) \quad \text{Eq. 56}$$

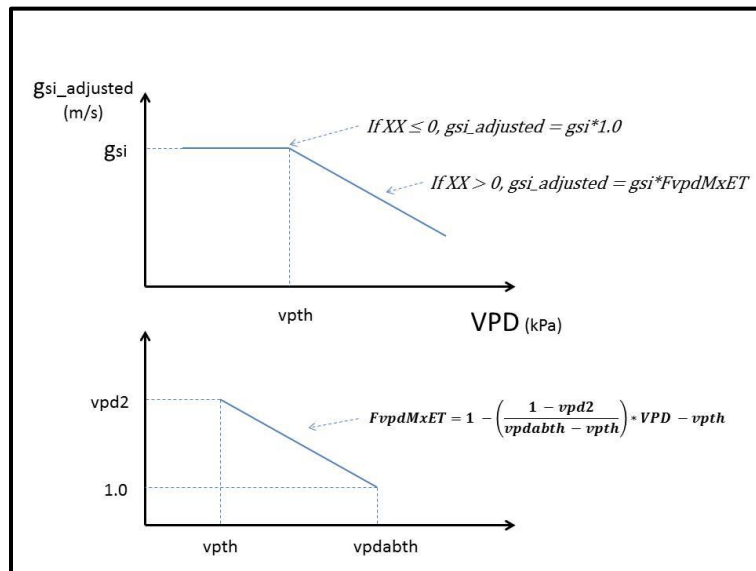


Figure 1. Stomatal conductivity adjustment for high vapor pressure

FvpdMxET: [SWAT 2005 Code]

gsi_adj: adjusted stomatal conductivity for high vapor pressure in m s^{-1}
[SWAT 2005 Code]

gsi: maximum stomatal conductance in m s^{-1}

bx: rate of decline in leaf conductance per unit increase in VPD ($\text{m s}^{-1} \text{kPa}^{-1}$)
[Eq. 2:2.2.16 in SWAT 2005]

vpd2: corresponding fraction of the maximum stomatal conductance at the value of VPD

vpdabth: value of VPD above *vpth*

vpth: threshold VPD above which the stomatal conductivity is adjusted in kPa

f) The canopy resistance for maximum plant ET is calculated by the following equation.

$$\text{CRMxET} = \frac{1.0}{(0.5 \cdot \text{LAI}) \cdot (\text{StomResp1} - \text{StomResp2} \cdot \frac{\text{CO}_2}{330})} \quad \text{Eq. 57}$$

Where:

CRMxET: Canopy resistance for maximum plant ET in s m^{-1}
[Eq. 2:2.2.15 in SWAT 2005]

LAI: Leaf area index of canopy

StomResp1: Stomatal response value 1 at elevated CO_2 concentration (C3/C4 crop parameter dimensionless)

StomResp2: Stomatal response value 2 at elevated CO_2 concentration (C3/C4 crop parameter dimensionless)

CO2: Carbon dioxide concentration in the atmosphere (ppm)

Canopy Interception

Canopy interception is the portion of rainfall that remains in the canopy and can not contribute to surface runoff or infiltration. This model allows the maximum amount of water that can be held in canopy storage to vary from day to day as a function of the leaf area index as follows:

$$\text{CanMxl} = \text{CanMx} * \frac{\text{LAI}}{\text{XLAI330}} \quad \text{Eq. 58}$$

Where:

CanMxl: maximum amount of water that can be trapped in the canopy on a given day in mm of H_2O [Eq. 2:2.2.1 in SWAT 2005]

CanMx: maximum amount of water that can be trapped in the canopy when the canopy is fully developed in mm of H_2O

LAI: leaf area index for a given day (dimensionless)

XLAI330: maximum leaf area index for the plant at 330 ppm of CO₂
(dimensionless)

When precipitation falls on any given day, the canopy storage is filled before any water is allowed to reach the ground and infiltrate or become surface runoff.

When rainfall is less than the difference between *CanMxl* and *CanStor*:

$$\mathbf{CanStor = CanStor + RF} \quad \mathbf{Eq. 59}$$

$$\mathbf{RF = 0} \quad \mathbf{Eq. 60}$$

Otherwise

$$\mathbf{CanStor = CanMxl} \quad \mathbf{Eq. 61}$$

$$\mathbf{RF = RF - (CanMxl - CanStor)}$$

Where

CanStor: amount of free water held in the canopy on a given day in mm
[Eq. 2:2.1.2 in SWAT 2005]

RF: rainfall on a given day in mm [Eq. 2:2.1.3 in SWAT 2005]

Once the potential evapotranspiration is determined, the actual evaporation is calculated. This model first evaporates any rainfall intercepted by the canopy. Next, the model calculates the maximum amount of transpiration and the maximum amount of soil evaporation.

The model removes as much water as possible from canopy storage when calculating actual evaporation. If potential evapotranspiration, *PETDAY*, is less than the amount of free water held in the canopy, *CanStor*, then

$$\mathbf{CanStor = (CanStor - PETDAY)} \quad \mathbf{Eq. 62}$$

$$\mathbf{CanET = PETDAY} \quad \mathbf{Eq. 63}$$

$$\mathbf{EPMax = 0} \quad \mathbf{Eq. 64}$$

$$\mathbf{ESMax = 0} \quad \mathbf{Eq. 65}$$

Otherwise

$$\mathbf{CanET = CanStor} \quad \mathbf{Eq. 66}$$

$$\mathbf{CanStor = 0} \quad \mathbf{Eq. 67}$$

Where:

CanET: Plant canopy evapotranspiration in mm

Potential Soil Evaporation

To calculate the potential soil evaporation (ESMax), PETDay from the Penman-Monteith method is used as follows:

$$\text{ESMax} = \text{PETDay} * e_{aj} \quad \text{Eq. 68}$$

$$\text{Eos1} = \text{PETDay} / (\text{ESMax} + \text{EPMax}) \quad \text{Eq. 69}$$

$$\text{Eos1} = \text{ESMax} * \text{Eos1} \quad \text{Eq. 70}$$

$$\text{ESMax} = \text{Min} (\text{ESMax}, \text{Eos1}) \quad \text{Eq. 71}$$

Where:

eaj: weighting factor for soil cover

To be sure that maximum plant and soil evapotranspiration do not exceed potential ET

$$\text{ESMax} = \text{PETDay} - \text{EPMax} \quad \text{if} \quad \text{PETDay} < \text{ESMax} + \text{EPMax} \quad \text{Eq. 72}$$

If: *PETDay* is less than *ESMax* + *EPMax*,

then:

$$\text{ESMax} = \text{PETDay} * \text{ESMax} / (\text{ESMax} + \text{EPMax}) \quad \text{Eq. 73}$$

$$\text{EPMax} = \text{PETDay} * \text{EPMax} / (\text{ESMax} + \text{EPMax}) \quad \text{Eq. 74}$$

Soil Water Balance and Actual Evapotranspiration

Soil Water Movement

Precipitation that is not intercepted by the canopy can become either surface runoff or infiltrate into the soil. Water in the soil exits the model domain through either transpiration, evaporation, or deep percolation out the bottom of the root zone.

Of these different pathways, plant uptake of water removes the majority of water that enters the soil profile. The potential plant uptake as a function of depth is calculated using:

$$UXLayers(i) = \frac{EPMax}{1 - \text{Exp}(-bw)} * (1 - \text{Exp}(-bw * \frac{TotLayDepth(i)}{RDepth})) \quad \text{Eq. 75}$$

Where:

UXLayers(i): potential transpiration from soil layer between the ground surface and the bottom of layer (i) in mm d⁻¹ [Eq. 5:2.2.1 in SWAT 2005]

bw: water-use distribution parameter (10 by default), dimensionless

TotLayDepth(i): distance from the soil surface to the bottom of layer (i) in mm

RDepth: depth of root development in the soil in mm

The potential water uptake from a particular soil layer can be calculated by solving the previous equation for the depth at the top and bottom of the soil layer and taking the difference between the values. Since root density is greatest near the soil surface and decreases with depth, the water uptake from the upper layer is assumed to be much greater than that in the lower layers. The water-use distribution parameter, *bw*, is set to 10 in PGM. With this value, 50% of the water uptake will occur in the upper 6% of the root zone.

As the water content of the soil decreases, the water in the soil is held more and more tightly by the soil particles. To reflect the effect this has on a plant's ability to extract water the following equation is used:

$$F(i) = \text{Exp}(5 * \frac{4 * SWLayer(i)}{AWCLayer(i)} - 1) \quad \text{if } SWLayer(i) < (AWCLayer(i) / 4) \quad \text{Eq. 76}$$

Otherwise

$$F(i) = 1.0 \quad \text{Eq. 77}$$

Where:

$F(i)$: water availability factor (dimensionless) for layer (i) [Eq. 5:2.2.4 in SWAT 2005]

$SWLayer(i)$: amount of water in the soil layer on a given day in mm

$AWCLayer(i)$: available water capacity for layer (i) in mm

$AWCLayer(i) = SWCFC(i) - SWCWP(i)$ [Eq. 5:2.2.6 in SWAT 2005]

$SWCFC(i)$: soil water content at field capacity for layer (i) (fraction)

$SWCWP(i)$: soil water content at wilting point for layer (i) (fraction)

The soil layers' thickness and the number of layers defined in the model is shown in Figure 2. There are 13 layers in total ($i = 13$). The top layer, which is the evaporation layer ($Z[1]$), is the only layer that is defined by the user in the interface.

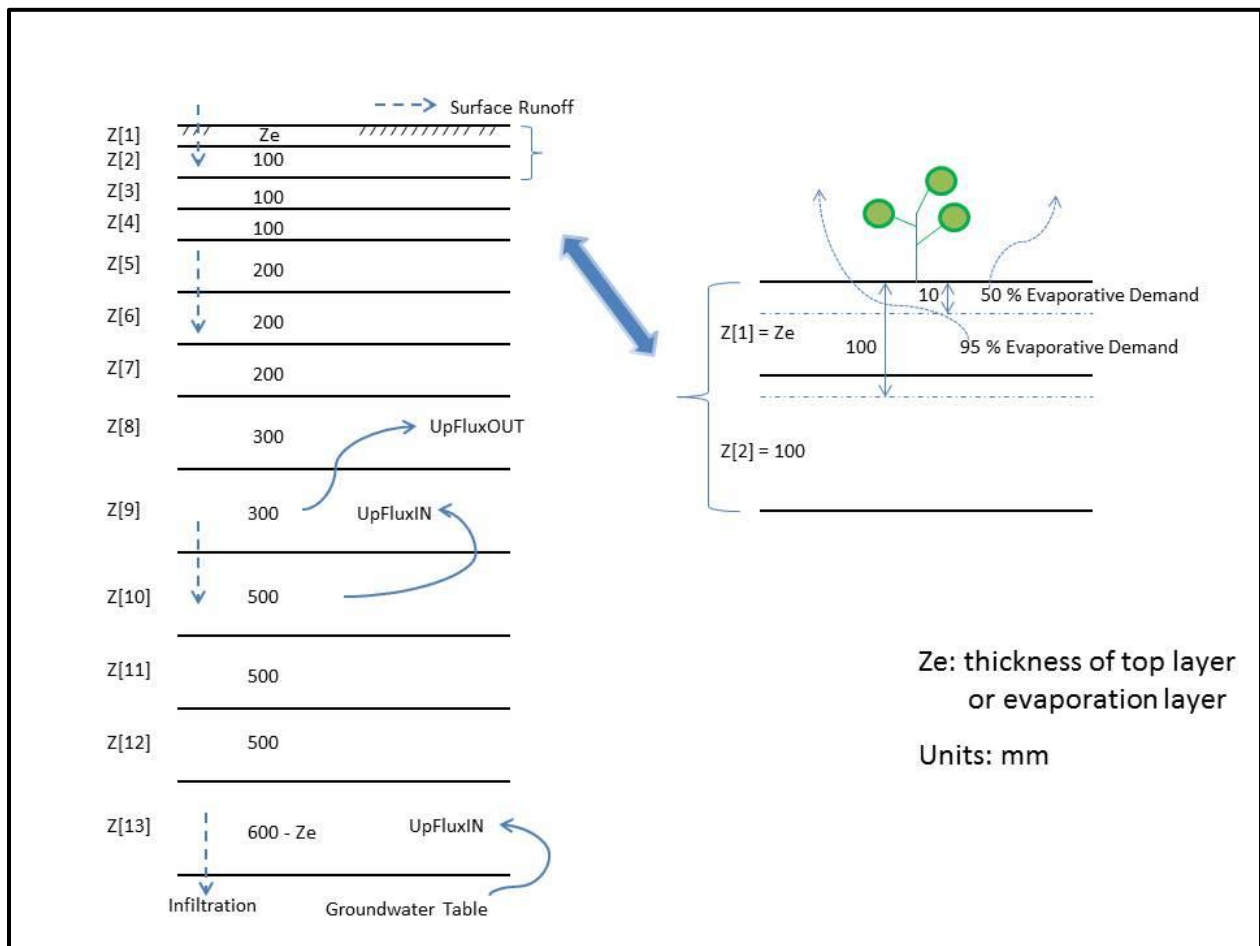


Figure 2. Soil layer profile

Once the potential water uptake and water availability factor have been obtained for soil water conditions, the actual amount of water uptake from the soil layer is calculated.

$$TALayer(i) = (UXLayer(i) + TRemain * epco) * F(i)$$

Eq. 78

$$\mathbf{TRemain} = \mathbf{UX} - \mathbf{TA}$$

Eq. 79

Where:

TALayer(i): actual water uptake from soil layer in mm [Eq. 5:2.2.3 in SWAT 2005]

TRemain: water uptake remaining in mm

epco: plant uptake compensation factor: 0 to 1.0 (dimensionless)

UX: potential water use rate for the whole soil profile in mm d⁻¹

TA: actual water uptake from the whole soil profile in mm d⁻¹

The plant uptake compensation factor (*epco*) allows plants to compensate for water deficiencies in dry layers by using water from other layers for soils with good rooting environments (*epco* near 1.0). However, compensation is reduced and finally is not allowed as *epco* approaches 0.0.

The total sum of the actual water uptake from all soil layers is the actual plant transpiration for the day. Once total actual plant transpiration is calculated, actual soil evaporation must be calculated. When an evaporation demand for soil exists, the model must first partition the evaporative demand between the different layers. The depth distribution used to determine the maximum amount of water allowed to be evaporated is:

$$\mathbf{EPlayer(i)} = \mathbf{ESMax} * \frac{\mathbf{TotLayDepth(i)}}{\mathbf{TotLayDepth(i)} + \mathbf{Exp(2.374 - 0.00713 * TotLayDepth(i))}} \quad \mathbf{Eq. 80}$$

Where:

EPLayer(i): potential evaporation demand for the soil between the soil surface and the bottom of layer (i) [Eq. 2:2.3.16 in SWAT 2005]

ESMax: potential soil evaporation in mm

TotLayDepth(i): total depth from the soil surface to bottom of layer (i) in mm

The coefficients in equation (80) were selected so that 50% of the evaporative demand is extracted from the top 10 mm of the soil and 95% of the evaporative demand is extracted from the top 100 mm of soil (Figure 2). The amount of evaporative demand for a particular soil layer is determined by taking the difference between the evaporative demands calculated at the upper and lower boundaries of the soil layer.

To reflect the decrease in soil water content in the evaporative water demand from drier soils, an evaporative water demand factor is determined based on the soil physical properties and estimated with the function:

$$\mathbf{EFactor(i)} = \mathbf{Min} \left(\mathbf{1.0}, \mathbf{Exp} \left(\mathbf{2.5} * \frac{\mathbf{SWLayer(i)} - \mathbf{SWCFC(i)}}{\mathbf{AWCLayer(i)}} \right) \right) \quad \mathbf{Eq. 81}$$

Where:

EFactor(i): evaporative water factor for layer (i) (dimensionless) [Eq. 2:2.3.18 in SWAT 2005]

SWLayer (i): amount of water in the soil layer on a given day in mm

SWCFC(i): soil water content at field capacity for layer (i) in mm

AWCLayer(i): available water capacity for layer (i) in mm

Once the potential evaporative soil demand has been obtained for soil water conditions, the actual amount of soil evaporation from the soil layer is calculated.

$$\mathbf{EALayer(i) = EPLayer(i) * EFactor(i)} \quad \mathbf{Eq. 82}$$

Where:

EALayer(i): actual amount of soil evaporation from the layer (i) in mm [Eq. 2:2.3.18 in SWAT 2005]

In addition to limiting the amount of water removed by evaporation in dry conditions, the model defines a maximum value of water that can be removed at any time. This maximum value is 80% of the plant available water on a given day where the plant available water is defined as the total water content of the soil layer minus the water content of the soil layer at wilting point.

$$\mathbf{EALayer'(i) = Min(EALayer(i), 0.8 * (SWLayer(i) - SWCWP(i)))} \quad \mathbf{Eq. 83}$$

Where:

EALayer'(i): amount of water removed from layer (i) by evaporation in mm [Eq. 2:2.3.20 in SWAT 2005]

The amount of water removed from soil layers is determined by taking the difference between the actual evaporative demands calculated at the upper and lower boundaries of the soil layers. Even further the model limits soil evaporation to some specific soil depth. The maximum soil depth from which evaporation is allowed to occur is set to 0.5 m.

Infiltration

Infiltration is determined using the Philip Equation. The root zone sorptivity is calculated if irrigation, rainfall or water ponding is greater than 0.0.

$$\mathbf{CapDrive = \frac{0.46*m+2.07*m^2+19.5*m^3}{1+4.7*m+16*m^2*1/\alpha}} \quad \mathbf{Eq. 84}$$

$$\mathbf{\beta = 1.3} \quad \mathbf{Eq. 85}$$

$$\mathbf{RZSorp} = (2 * (\mathbf{SWCS} - \mathbf{ThRZ}) * \mathbf{Ksat} * \frac{\mathbf{CapDrive}}{\beta})^{1/2} \quad \mathbf{Eq. 86}$$

$$\mathbf{MaxInfil} = (\mathbf{RZSorp} * \sqrt{\mathbf{InfilEnd}} + \mathbf{Ksat} * \mathbf{InfilEnd}) - (\mathbf{RZSorp} * \sqrt{\mathbf{InfilStart}} + \mathbf{Ksat} * \mathbf{InfilStart}) \quad \mathbf{Eq. 87}$$

Where:

CapDrive: capillary drive

$m = 1 - 1/n$

n: van Genuchten parameter

α : inverse of the air-entry value (bubbling pressure)

β : assumed to be 1.3

SWCS: soil water content at saturation, dimensionless

ThRZ: soil water content in root zone, dimensionless

Ksat: saturated hydraulic conductivity in length/time

RZSorp: The root zone sorptivity in length/time

MaxInfil: maximum infiltration rate in length/time [PGM Internal Code]

InfilEnd: time of infiltration end

InfilStart: time of infiltration start

Upflux coming into layer (i) from underneath layer (Figure 2) is calculated with the function:

$$\mathbf{UpFlux(i)} = \mathbf{KUpFlux(i)} * \frac{(\psi_{\mathbf{A}+\mathbf{CLayDepth(i-1)}} - \psi_{\mathbf{B}+\mathbf{CLayDepth(i)}})}{\mathbf{CLayDepth(i-1)} - \mathbf{CLayDepth(i)}} \quad \mathbf{Eq. 88}$$

Where:

UpFlux(i): Upflux coming into layer (i) in mm

KUpFlux(i): upflux hydraulic conductivity in mm s⁻¹

$\psi_{\mathbf{A}}$: Pressure head at point A

$\psi_{\mathbf{B}}$: Pressure head at point B

CLayDepth(i): center layer depth point for layer (i) in mm

The main assumption is that $\psi_{\mathbf{B}}$ is greater than $\psi_{\mathbf{A}}$ for upflux to happen.

If the depth of the center of layer (i) is below the groundwater table depth, the soil water content is adjusted to saturation.

$$\mathbf{Th(i)} = \mathbf{SWCS(i)} \quad \mathbf{if} \quad \mathbf{CLayDepth(i)} > \mathbf{WTDepth} \quad \mathbf{Eq. 89}$$

Where:

WTDepth: water table depth in mm [PGM Internal Code]

The pond mass balance is computed only for the top layer when there is water ponding.

If the potential soil evaporation (ESMax) is greater or equal than total depth of applied water:

$$\begin{aligned} \text{Infil}(i) &= 0.0 && \text{Eq. 90} \\ \text{EvapRemain} &= \text{ESMax} - \text{DSP} + \text{TWDAL} && \text{Eq. 91} \\ \text{SurfEvap} &= \text{DSP} + \text{TWDAL} && \text{Eq. 92} \\ \text{DSP} &= 0.0 && \text{Eq. 93} \\ \text{SWRO} &= 0.0 && \text{Eq. 94} \end{aligned}$$

If the potential soil evaporation (ESMax) is smaller than total applied water depth:

$$\begin{aligned} \text{Infil}(i) &= \text{Min}(\text{DSP} + \text{TWDAL} - \text{ESMax}, \text{MaxInfil}) && \text{Eq. 95} \\ \text{EvapRemain} &= 0.0 && \text{Eq. 96} \\ \text{SurfEvap} &= \text{ESMax} && \text{Eq. 97} \\ \text{DSP} &= \text{Min}(\text{MaxPond}, \text{DSP} + \text{TWDAL} - \text{ESMax} - \text{Infil}(i)) && \text{Eq. 98} \\ \text{SWRO} &= \text{DSP} + \text{TWDAL} - \text{ESMax} - \text{Infil}(i) - \text{MaxPond} && \text{Eq. 99} \end{aligned}$$

Where:

Infil(i): infiltration into soil layer (i) in mm
EvapRemain: evaporation remain in mm
SurfEvap: surface evaporation in mm
DSP: depth of surface ponding in mm
SWRO: surface water runoff in mm
MaxPond: maximum depth of surface ponding in mm
TWDAL: total water depth applied to land in mm

If surface ponding is not present, runoff may still occur. In this case the model first determines if the total applied water depth is greater than the maximum infiltration and the maximum ponding depth.

$$\begin{aligned} \text{SWRO} &= \text{TWDAL} - (\text{MaxInfil} + \text{MaxPond}) && \text{Eq. 100} \\ \text{DSP} &= \text{MaxPond} && \text{Eq. 101} \\ \text{Infil}(i) &= \text{MaxInfil} && \text{Eq. 102} \\ \text{EvapRemain} &= \text{EA} + \text{TA} && \text{Eq. 103} \\ \text{SurfEvap} &= 0.0 && \text{Eq. 104} \end{aligned}$$

If the opposite condition is reached, the model uses the following relationships.

$$\begin{aligned} \text{SWRO} &= 0.0 && \text{Eq. 105} \\ \text{DSP} &= \text{TWDAL} - \text{MaxInfil} && \text{Eq. 106} \\ \text{Infil}(i) &= \text{MaxInfil} && \text{Eq. 107} \\ \text{EvapRemain} &= \text{EA} + \text{TA} && \text{Eq. 108} \\ \text{SurfEvap} &= 0.0 && \text{Eq. 109} \end{aligned}$$

Where:

MaxPond: maximum ponding depth in mm

EA: actual soil evaporation in mm
TA: actual plant transpiration in mm

When the total applied water depth is greater than the maximum infiltration rate, the model determines the following:

$$\begin{aligned} \mathbf{SWRO} &= \mathbf{0.0} && \mathbf{Eq. 110} \\ \mathbf{DSP} &= \mathbf{0.0} && \mathbf{Eq. 111} \\ \mathbf{Infilt(i)} &= \mathbf{TWDAL} && \mathbf{Eq. 112} \\ \mathbf{EvapRemain} &= \mathbf{EA} && \mathbf{Eq. 113} \\ \mathbf{SurfEvap} &= \mathbf{0.0} && \mathbf{Eq. 114} \end{aligned}$$

And finally when there is no water applied at all, PGM determines the following:

$$\begin{aligned} \mathbf{SWRO} &= \mathbf{0.0} && \mathbf{Eq. 115} \\ \mathbf{DSP} &= \mathbf{0.0} && \mathbf{Eq. 116} \\ \mathbf{Infilt(i)} &= \mathbf{0.0} && \mathbf{Eq. 117} \\ \mathbf{EvapRemain} &= \mathbf{EA} && \mathbf{Eq. 118} \\ \mathbf{SurfEvap} &= \mathbf{0.0} && \mathbf{Eq. 119} \end{aligned}$$

The following step is to compute the soil layer water mass balance. There are two potential conditions. The first condition is when there is infiltration at the soil surface.

For soil layers below the groundwater table:

If *CLayDepth(i)* is greater than *WTDepth*:

$$\mathbf{Infilt(i)} = \mathbf{Min(Ksat, infilt(i) - UpFluxOut(i))} \quad \mathbf{Eq. 120}$$

For cases in which there is infiltration:

For the case where infiltration fills the soil in excess of saturation:

$$\mathbf{Infilt(i)} = (\mathbf{Infilt(i - 1)} + \mathbf{UpFluxIn(i - 1)} - \mathbf{UpFluxOut(i - 1)} + \mathbf{Th(i - 1)} - \mathbf{evap} - \mathbf{TALayer(i - 1)} - \mathbf{SWCS(i - 1)}) \quad \mathbf{Eq. 121}$$

For the case in which infiltration fills soil between field capacity and saturation:

$$\mathbf{Infilt(i)} = (\mathbf{Infilt(i - 1)} + \mathbf{UpFluxIn(i - 1)} - \mathbf{UpFluxOut(i - 1)} + \mathbf{Th(i - 1)} - \mathbf{evap} - \mathbf{TALayer(i - 1)} - \mathbf{SWCFC(i - 1)}) * (\mathbf{1} - \mathbf{DCF}) \quad \mathbf{Eq. 122}$$

For the case in which irrigation fills soil to less than field capacity:

$$\mathbf{Infilt(i)} = \mathbf{0.0} \quad \mathbf{Eq. 123}$$

The second condition is for when there is no infiltration at the ground surface:

For the case when the soil water content is in excess of saturation:

$$\mathbf{Infilt(i) = \text{Min}(K_{sat}, (SWCS(i - 1) - SWCFC(i - 1)) * (1 - DCF))}$$

Eq. 124

For the case in which the soil water content is between field capacity and saturation:

$$\mathbf{Infilt(i) = \text{Min}(K_{sat}, (Infilt(i - 1) + UpFluxIn(i - 1) - UpFluxOut(i - 1) + Th(i - 1) - evap - TAlayer(i - 1) - SWCFC(i - 1)) * (1 - DCF))}$$

Eq. 125

For the case in which irrigation fills soil to less than field capacity:

$$\mathbf{Infilt(i) = 0.0}$$

Eq. 126

Where:

Infilt(i): infiltration into soil layer (i) in mm

UpFluxIn(i): upflux going in to layer (i) in mm

UpFluxOut(i): upflux going out from layer (i) in mm

Th(i): soil water content for layer (i) in mm

evap: soil evaporation in mm

TAlayer(i): actual plant water uptake from soil layer (i) in mm

SWCS: soil water content at saturation in mm

SWCFC: soil water content at field capacity in mm

DCF: soil water content decline factor (dimensionless) [Eq. 2:3.2.3 in SWAT 2005]

$$\mathbf{DCF = 1 - \text{Exp}(-\frac{1}{TT})}$$

Eq. 127

$$\mathbf{TT = \frac{SWCS - SWCFC}{K_{sat}}}$$

Eq. 128

$$\mathbf{evap = \text{Min}(EALayer(i), EvapRemain)}$$

Eq. 129

Where:

TT: travel time for percolation (hrs) [Eq. 2:3.2.4 in SWAT 2005]

evap: evaporation

The model checks that plant transpiration and soil evaporation won't reduce the soil water content below wilting point. If the soil water available for transpiration and evaporation is less than what is demanded, both evaporation and transpiration are reduced using relative weights as follows:

$$\mathbf{EFrac} = \frac{\mathbf{evap}}{\mathbf{evap} + \mathbf{TALayer}(i)} \quad \mathbf{Eq. 130}$$

$$\mathbf{TFrac} = \frac{\mathbf{TALayer}(i)}{\mathbf{TALayer}(i) + \mathbf{evap}} \quad \mathbf{Eq. 131}$$

$$\mathbf{TALayer}(i) = \mathbf{TFrac} * \left((\mathbf{Th}(i) - \mathbf{SWCWP}(i)) + \mathbf{Infilt}(i) + \mathbf{UpFluxIN}(i) - \mathbf{UpFluxOUT}(i) \right) \quad \mathbf{Eq. 132}$$

$$\mathbf{EALayer}(i) = \mathbf{EFrac} * \left((\mathbf{Th}(i) - \mathbf{SWCWP}(i)) + \mathbf{Infilt}(i) + \mathbf{UpFluxIN}(i) - \mathbf{UpFluxOUT}(i) \right) \quad \mathbf{Eq. 133}$$

The model may allow evaporation to decrease the soil water content below wilting point.

$$\mathbf{EALayer}(i) = \mathbf{EALayer}(i) + \mathbf{Min}(\mathbf{EvapLeft}, \mathbf{P5} * \mathbf{SWCWP}(i)) \quad \mathbf{Eq. 134}$$

Where:

EFrac: fraction of evaporation to evaporate (dimensionless) [PGM Internal Code]

TFrac: fraction of transpiration to transpire (dimensionless) [PGM Internal Code]

EvapLeft: evaporation that was not met in mm [PGM Internal Code]

P5: maximum water content that can be removed below wilting point (0.0 ≤ P5 ≤ 1) in the top 0.5 m of soil and it is set to 1.0 below 0.5 m (dimensionless)

Thus, model can be adjusted to allow the top 0.5 m of soil to dry down to any fraction of wilting point.

Finally the model recalculates the new soil water content by doing a soil water mass balance for each soil layer.

$$\mathbf{Th}(i - 1) = \mathbf{Th}(i - 1) + \mathbf{Infilt}(i - 1) + \mathbf{UpFluxIn}(i - 1) - \mathbf{UpFluxOut}(i - 1) + \mathbf{EALayer}(i - 1) - \mathbf{TALayer}(i - 1) - \mathbf{Infilt}(i) \quad \mathbf{Eq. 135}$$

Also the model checks that no layer has a water content greater than saturation. If such a condition exists, then the water in excess of saturation is transferred to the layer above.

$$\mathbf{Transfer}(i) = \mathbf{Th}(i) - \mathbf{SWCS}(i) \quad \mathbf{if} \quad \mathbf{Th}(i) > \mathbf{SWCS}(i) \quad \mathbf{Eq. 136}$$

$$\mathbf{Th}(i) = \mathbf{SWCS}(i) \quad \mathbf{if} \quad \mathbf{Th}(i) > \mathbf{SWCS}(i) \quad \mathbf{Eq. 137}$$

$$\mathbf{Transfer}(i) = \mathbf{0.0} \quad \mathbf{if} \quad \mathbf{Th}(i) \leq \mathbf{SWCS}(i) \quad \mathbf{Eq. 138}$$

If there is a correction for excess water in the top layer, the surface runoff is adjusted.

$$\mathbf{SWRO} = \mathbf{SWRO} + \mathbf{Transfer}(i) \quad \mathbf{Eq. 139}$$

Where:

Transfer(i): excess water transfer to layer (i) in mm

SWCS(i): soil water content at saturation in layer (i) in mm

SWRO: surface runoff in mm

Crop Growth

Crop growth is simulated with a single model using different parameters for different crop types. Due to the similarities with the APEX and SWAT models, the model can be parameterized using the databases provided with those models. The growth season for annual crops can be initiated at a user specified planting date or once a user specified number of heat units has accumulated. Harvest can be specified as a date or as a function of heat unit accumulation. Perennial crops initiate growth once the daily average air temperature exceeds the crop specific base temperature.

Phenological development of the crop is based on daily heat unit accumulation. It is computed using the equation:

$$\mathbf{HU = 0.5 * (TMX + TMN) - TBSC} \quad \mathbf{HU > 0} \quad \mathbf{Eq. 140}$$

Where:

HU: number of heat units accumulated during a day [Eq. 5:1.1.1 in SWAT 2005]

TMX: maximum temperatures for the day in °C

TMN: minimum temperatures for the day in °C

TBSC: crop-specific base temperature of all variables in °C (no growth occurs at or below TBSC)

A heat unit index is calculated by dividing the accumulated heat units by the total required for maturity ($HUI = \text{acc HU} / \text{potential HU}$). The HUI ranges from 0.0 at germination to 1.0 at harvest. The timing of harvest, leaf area growth and senescence, and partition of dry matter among roots, shoots, and economic yield are affected by HUI.

Potential Growth

Potential growth is calculated using the following formula. Potential growth is the growth that can occur if there is no temperature, water, or nutrient stress. In this version of the model, only temperature and water stress are simulated.

$$\mathbf{Bio = 0.001 * PAR * (RUE - WAVP * X1)} \quad \mathbf{Eq. 141}$$

Where:

Bio: daily potential increase in biomass in $\text{t ha}^{-1} \text{d}^{-1}$ [Eq. 275 in APEX 2008]

PAR: intercepted photosynthetic active radiation in $\text{MJ m}^{-2} \text{d}^{-1}$

RUE: radiation-use efficiency factor for converting energy to biomass ($\text{kg ha}^{-1} / (\text{MJ m}^{-2})$)

WAVP: crop parameter relating RUE and VPD

$$\text{PAR} = 0.5 * \text{RA} * (1.0 - \exp(-0.65 * \text{LAI})) \quad \text{Eq. 142}$$

Where:

PAR: [Eq. 5:2.1.1 in SWAT 2005]

RA: solar radiation in MJ m⁻² d⁻¹

LAI: leaf area index (dimensionless)

Constant 0.5: used to convert solar radiation to photosynthetically active radiation

Constant 0.65: extinction coefficient

$$\text{RUE} = \frac{100 * \text{CO}_2}{\text{CO}_2 + \exp(\text{bc1} - \text{bc2} * \text{CO}_2)} \quad \text{Eq. 143}$$

Where:

RUE: [Eq. 5:2.1.4 in SWAT 2005]

CO₂: atmospheric CO₂ level in ppm

bc1, bc2: crop parameters determined from two input points on the RUE-CO₂ curve (Stockle et al., 1992)

$$X1 = \max(0.0, \text{VPD} - \text{VPD}_{th}) \quad \text{Eq. 144}$$

Where:

X1: [Eq. 275b in APEX 2008]

VPD: vapor pressure deficit in kPa

VPD_{th}: threshold vpd (default = 1.0)

LAI is simulated as a function of heat units, crop stress, and crop development stage. From emergence to the start of leaf decline, LAI is estimated with the equations:

$$\text{LAI} = \text{LAI}_0 + d\text{HUF} * \text{XLAI} * (1.0 - \text{Exp}(5.0 * \text{LAI}_0 - \text{XLAI}) * \text{sqrt}(\text{REG})) \quad \text{Eq. 145}$$

$$\text{HUF} = \frac{\text{HUI}}{\text{HUI} + \exp(\ell(1) - \ell(2) * \text{HUI})} \quad \text{Eq. 146}$$

Where:

LAI: leaf area index value of the crop at the end of the day, dimensionless
[Eq. 5:2.1.16 in SWAT 2005]

LAI₀: leaf area index value of the crop at the beginning day, dimensionless

dHUF: daily change in HUF, dimensionless

HUF: heat unit factor, dimensionless [Eq. 5:2.1.10 in SWAT 2005]

XLAI: maximum leaf area index of the crop, dimensionless

REG: value of the minimum crop stress factor, dimensionless

HUI: heat unit index (0 at planting to 1 at physiological maturity) of the crop, dimensionless

$\ell 1$ and $\ell 2$ coefficients: crop parameters relating HUF and HUI for crop

$$\mathbf{HUI} = \frac{\mathbf{Acc\ daily\ HU}}{\mathbf{Potential\ HU}} \quad \mathbf{Eq. 147}$$

Where:

HUI: [Eq. 5:2.1.11 in SWAT 2005]

Acc Daily HU: Cumulative heat units

Potential HU: Number of heat units required to reach maturity

From the start of leaf decline to the end of the growing season, LAI is estimated with the equation:

$$\mathbf{LAI} = \mathbf{XLAI} * \left[\frac{\mathbf{1.0 - HUI}}{(\mathbf{1.0 - HUI}_D)} \right] \quad \mathbf{Eq. 148}$$

Where:

LAI: [Eq. 5:2.1.19 in SWAT 2005]

HUI_D: value of HUI when LAI starts declining

Crop height is estimated with the relationship:

$$\mathbf{CPHT} = \mathbf{HMX} * \mathbf{sqrt(HUF)} \quad \mathbf{Eq. 149}$$

Where:

CHT: crop height in m [Eq. 5:2.1.14 in SWAT 2005]

HMX: maximum height for crop

HUF: heat unit factor

In the model it is assumed that the portion of total biomass production allocated to the roots declines from a value of 0.4 at germination to 0.2 at maturity. The root allocation fraction is computed with the equation:

$$\mathbf{Fr_{root}} = \mathbf{0.40 - 0.20 * HUI} \quad \mathbf{Eq. 150}$$

Where:

Fr_{root}: Fraction of total biomass partitioned to roots on a given day in the growing season, [Eq. 5:2.1.21 in SWAT 2005]

HUI: Fraction of potential heat units accumulated for the plant on a given day in the growing season

The potential above-ground biomass is estimated as a fraction of the total crop biomass production that considers the fraction of biomass partitioned to the root system.

$$\mathbf{Bio}_{\text{above}} = (1.0 - Fr_{\text{root}}) * \mathbf{Bio} \quad \mathbf{Eq. 151}$$

Where:

Bio_{above} : potential aboveground biomass on a given day in $t \text{ ha}^{-1}$, [Eq. 5:2.4.4 in SWAT 2005]

Rooting depth is simulated as a function of heat units and potential root zone depth:

$$\mathbf{RD} = \mathbf{min}(2.5 * \mathbf{RDMX} * \mathbf{HUI}, \mathbf{RDMX}, \mathbf{RZ}) \quad \mathbf{Eq. 152}$$

Where:

RD : root depth in m for crop [Eq. 5:2.1.23 in SWAT 2005]

$RDMX$: maximum root depth in m for crop

HUI : heat unit index of the crop

RZ : soil profile depth in m

Growth Constraints

In the model growth can be limited by water and/or temperature stress.

Water Stress Factor:

The water stress factor is computed by considering the potential transpiration which is a function of the leaf area, stomatal conductance, and atmospheric conditions (EPMax). This value is compared to the moisture constrained transpiration (TALayers) that accounts for the moisture status of the soil.

$$\mathbf{WS} = \frac{\mathbf{TALayers}}{\mathbf{EPMax}} \quad \mathbf{Eq. 153}$$

Where:

WS : water stress factor for a specific crop (dimensionless) [Eq. 5:3.1.1 in SWAT 2005]

$TALayers$: actual plant water uptake from soil layers in mm d^{-1}

$EPMax$: maximum plant transpiration in mm d^{-1}

Temperature Stress Factor:

The plant temperature stress is estimated with the equation

$$\mathbf{TS = Exp (-0.1054 * RTO)} \quad \mathbf{RTO \leq 200 \text{ and } TGX > 0.0} \quad \mathbf{Eq. 154}$$

$$\mathbf{TS = 0.0} \quad \mathbf{RTO > 200 \text{ or } TGX \leq 0.0} \quad \mathbf{Eq. 155}$$

$$\mathbf{RTO = \frac{TOPC - TX}{2 * TGX}} \quad \mathbf{Eq. 156}$$

$$\mathbf{TGX = TX - TBSC} \quad \mathbf{TX \leq TOPC} \quad \mathbf{Eq. 157}$$

Where:

TS: plant temperature stress factor (dimensionless) [Eq. 5:3.1.2 to Eq. 5:3.1.5 in SWAT 2005]

TX: average daily air temperature in °C

TBSC: base temperature for corresponding crop in °C

TOPC: optimal temperature for corresponding crop in °C

Finally, the plant stress factor is determined as the lowest value of the WS and TS stress factors.

$$\mathbf{REG = Min (TS, WS)} \quad \mathbf{Eq. 158}$$

Where:

REG: plant stress factor due to *TS* and *WS*, dimensionless

Actual Growth

Actual growth is calculated as a function of the potential growth and the plant stress factor:

$$\mathbf{ActBio = Bio * REG} \quad \mathbf{Eq. 159}$$

Where:

ActBio: actual plant biomass on a given day in t ha⁻¹ [Eq. 5:3.2.1 in SWAT 2005]

Bio: potential increase in biomass in t ha⁻¹ d⁻¹

REG: plant stress factor due to *TS* and *WS*, dimensionless

For the above-ground biomass the following equation is used.

$$\mathbf{ActBio_{above} = Bio_{above} * REG} \quad \mathbf{Eq. 160}$$

Where:

ActBio_{above}: actual above-ground biomass on a given day in t ha^{-1} [Eq. 5:3.2.1 in
SWAT 2005]

Bio_{above}: potential increase in above-ground biomass in $\text{t ha}^{-1} \text{d}^{-1}$

Economic Yield

In this model, economic yield is calculated using a harvest index. The harvest index specifies the portion of the plant mass that is harvested. This value is relatively stable for a range of plant stress (SWAT 2005):

Harvest Index is calculated for each day of the plant's growing season using the relationship:

$$HI = HI_{opt} * \frac{100 * HUI}{(100 * HUI + \exp(11.11 - 10 * HUD))} \quad \text{Eq. 161}$$

Where:

HI: Potential harvest index on the day of harvest, dimensionless [Eq. 5:2.4.1 in SWAT 2005]

HI_{opt}: potential harvest index for the plant at maturity given ideal growing conditions

HUI: heat unit index (fraction of potential heat units accumulated for the plant on a given day in the growing season)

The potential crop yield is calculated as:

$$yld = Bio_{above} * HI \quad \text{when } HI \leq 1.00 \quad \text{Eq. 162}$$

$$yld = Bio * \left(1 - \frac{1}{(1 + HI)} \right) \quad \text{when } HI \geq 1.00 \quad \text{Eq. 163}$$

Where:

yld: crop yield in t ha⁻¹ [Eq. 5:2.4.2 & Eq. 5:2.4.3 in SWAT 2005]

Bio_{above}: above-ground biomass on the day of harvest t ha⁻¹

HI: harvest index on the day of harvest

Actual Crop Yield

In this model an actual harvest index is calculated during the second half of the crop growth season. This actual harvest index accounts for the impact of cumulative water stress on crop yield.

$$YLD_{Actual} = HI_{Actual} * ActBio_{Above} \quad \text{Eq. 164}$$

$$HI_{Actual} = (HI - HI_{Min}) \frac{WS}{WS + \exp(6.13 - 0.0883 * WS)} + HI_{Min} \quad \text{Eq. 165}$$

where:

YLD_{Actual} : actual crop yield in t ha⁻¹ [Eq. 281 in APEX 2008]

HI_{Actual} : actual harvest index used to estimate crop yield, dimensionless [Eq. 5:3.3.1 in SWAT 2005]

HI : potential harvest index on the day of harvest, dimensionless

HI_{Min} : minimum harvest index for a specific crop, dimensionless

Rice Specific Algorithms

Accurately reproducing water management practices can be one of the most complicated portions of modeling. Because water management affects the hydrologic balance, it is critical that the model is able to accommodate management practices like those used in rice production. In this section the rice ponding algorithm is described starting with pond evaporation.

The volume of water lost to evaporation from the pond is calculated using a factor, n (0.875), for free surface evaporation. The factor 0.875 is the ratio of the crop coefficient found in FAO56 for open water less than 2 m deep (1.05) and the conversion from the alfalfa reference (PETDay) to the short grass reference (1.2). The total potential evaporation is then further reduced by the transpiration (EPMax) which accounts for the growth of the rice crop:

$$EPond = n * PETDay - EPMax \quad \text{Eq. 166}$$

where:

$EPond$: evaporation from water surface in mm

n : evaporation coefficient (0.875), dimensionless

The volume of water lost to evaporation from a rice field:

$$TaRice = EPMax + CanET \quad \text{Eq. 167}$$

where:

$TaRice$: transpiration from rice in mm

Rice Ponding

Rice ponding is controlled by parameters that specify the depth of ponding required during various stages of rice crop development. The timing of ponding depth requirements is specified either using heat units or calendar dates. If using heat units to determine planting date and development (HU-HU):

$$MaxPondD = MaxPondD_1 \quad \text{if } CumHU < Pre_1 \quad \text{Eq. 168}$$

$$MaxPondD = MaxPondD_2 \quad \text{if } Pre_1 \leq CumHU < Pre_2 \quad \text{Eq. 169}$$

$$MaxPondD = MaxPondD_3 \quad \text{if } Pre_2 \leq CumHU < Initial \quad \text{Eq. 170}$$

$$MaxPondD = ((MaxPondD_4 - (MaxPondD_3 + 0.0833)) * (\frac{CropHU}{Develop})) + MaxPondD_3 + 0.0833$$

If JulianDay ≥ Initial and CropHU < Develop Eq. 171

$$MaxPondD = ((MaxPondD_5 - MaxPondD_4) * (\frac{CropHU - Develop}{Mid - Develop})) + MaxPondD_4$$

If Develop ≤ CropHU < Mid Eq. 172

$$MaxPondD = ((MaxPondD_6 - MaxPondD_5) * (\frac{CropHU - Mid}{Late - Mid})) + MaxPondD_5$$

If Mid ≤ CropHU < Late Eq. 173

$$MaxPondD = ((MaxPondD_7 - MaxPondD_6) * (\frac{CropHU - Late}{EndLate - Late})) + MaxPondD_6$$

If Late ≤ CropHU < EndLate Eq. 174

where:

MaxPondD : maximum ponding depth in mm [PGM Internal Code]

MaxPondD₁ : maximum ponding depth before pre-flooding stage in ft

MaxPondD₂ : maximum ponding depth during pre-flooding stage in ft

MaxPondD₃ : maximum ponding depth during non-flooding stage in ft

MaxPondD₄ : maximum ponding depth during initial stage in ft

MaxPondD₅ : maximum ponding depth during develop stage in ft

MaxPondD₆ : maximum ponding depth during mid-stage in ft

MaxPondD₇ : maximum ponding depth during late stage in ft

MaxPondD₈ : maximum ponding depth for endlate stage in ft

Pre_1 : heat units required for pre-stage_1 of flooding since January 1, dimensionless

Pre_2 : heat units required for pre-stage_2 of non-flooding since January 1, dimensionless

Initial : heat units required for initial growing stage or planting date heat units threshold since January 1, dimensionless

Develop : heat units required for develop growing stage since planting day, dimensionless

Mid : heat units required for mid growing stage since planting day, dimensionless

Late : heat units required for late growing stage since planting day, dimensionless

EndLate : heat units required for end growing stage since planting day, dimensionless

CumHU : accumulated heat units since January 1, calculated with a base temperature of 0.0 degrees Celsius.

For the FIX-HU approach the previous algorithms from Eq. 168 to Eq. 174 apply as well. The only difference is how the timing of the initial stages is determined. In the FIX-HU approach the stages prior to planting are fixed and determined based on Julian Days and then heat units is the driver for the development stages. The way that these stages are determined for the FIX-HU approach is described below:

Pre_1 : julian day for pre-stage_1 of flooding, dimensionless
Pre_2 : julian day for pre-stage_2 of non-flooding, dimensionless
Initial : julian day for initial growing stage or planting date, dimensionless
Develop : heat units required as a fraction of PHU for develop growing stage since planting day, dimensionless
Mid : heat units required as a fraction of PHU for mid growing stage since planting day, dimensionless
Late : heat units required as a fraction of PHU for late growing stage since planting day, dimensionless
EndLate : heat units required as a fraction of PHU for end growing stage since planting day, dimensionless

To better understand the different ponding depths and growing stages for the complete rice growing season a scheme of them is shown in Figure 3. As it is observed in the scheme, the different stages can be determined based on heat units (HU-HU approach) or a combination of Julian days and heat units (Fix-HU approach). Both approaches determine when a specific stage starts and ends.

1. Initially there is a five-days flood-up stage (*Pre_1*) where a ponding depth of 3 inches is reached.
2. A non-ponding period of 10-days follows the flood-up stage (*Pre_2*).
3. Seeding happens at the beginning of the Initial stage (May 1) with a gradually flood-up period until 5 inches of ponding is reached by the beginning of the Develop stage.
4. Gradually flood-up continues up to 8 inches of ponding at the beginning of Mid stage.
5. The 8 inches ponding stays until Late stage is reached.
6. From Late stage to *EndLate* stage the rice fields are gradually lowered down to having non-ponding depth. During this stage there is not irrigation application anymore to the rice fields. Harvest may happen any time after the *EndLate* stage.

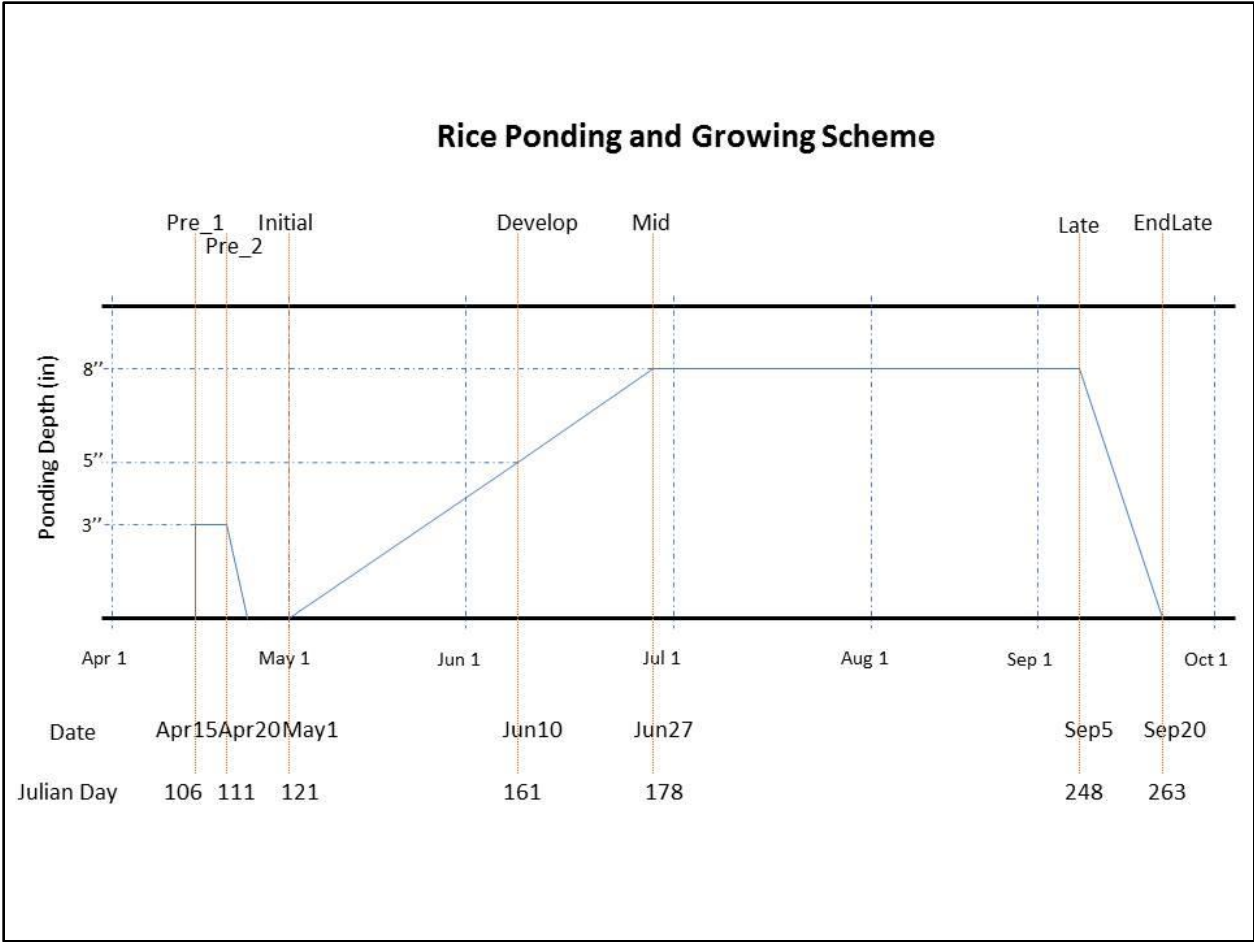


Figure 3. Representation of Rice fields' ponding and Rice growing season

Deciduous Specific Algorithms

Deciduous crops are those that lose all their leaves in the fall and go dormant for a period of time. Examples are: Almonds, Apples, Orchards, and Vineyards. For deciduous crops a couple of specific conditions apply for crop management; the time when irrigation should be stopped and the time when leaves start falling. The first one defines when irrigation ceases. This condition is needed because after harvest happens, deciduous crops are typically given reduced irrigation that eventually ceases as the weather cools. To determine the exact day when irrigation should be stop for each year, a temperature threshold is used. If there are three days with an average temperature less than the threshold, then irrigation is stopped.

$$\begin{aligned} \text{If } & \text{MinTemp} < \text{StopIrrMinTemp} && \text{Eq. 175} \\ & \text{JulianDay} > 274 \text{ (Oct 1)} \\ & \text{NumDaysTempOct1} > 3 \end{aligned}$$

Then

$$\text{StopDecidIrrig} = \text{true} \quad \text{Eq. 176}$$

Where

MinTemp: minimum temperature on a specific day in °C

StopIrrMinTemp: temperature threshold for which irrigation stops in °C

NumDaysTempOct1: number of days with minimum temperatures lower than the temperature threshold, dimensionless

StopDecidIrrig: flag that indicates that irrigation must be stopped

A similar approach is used to determine the day when the fall starts and consequently deciduous trees start losing their leaves. Deciduous evapotranspiration is maintained until these conditions are satisfied.

$$\begin{aligned} \text{If } & \text{MinTemp} < \text{FallLeavesMinTemp} && \text{Eq. 177} \\ & \text{JulianDay} > 305 \text{ (Nov 1)} \\ & \text{NumDaysTempNov1} > 3 \end{aligned}$$

Then

$$\text{StartFall} = \text{true} \quad \text{Eq. 178}$$

Where

FallLeavesMinTemp: temperature threshold for which fall starts in °C

NumDaysTempNov1: number of days with minimum temperatures lower than the temperature threshold, dimensionless

StartFall: flag that indicates that fall starts and leaves fall [PGM Internal Code]

Finally the Julian Days may vary depending on the crop to be modeled.

Perennials Specific Algorithms

Perennial non-deciduous crops are treated in some specific ways as well. Examples of crops that fall under this classification are: Alfalfa, Pasture and Urban Lawn. A specific number of cuttings are defined for each crop type. For the Alfalfa crop up to 7 fixed cuttings are defined, and they are scheduled to happen each year based on this regular defined schedule and using Julian Days as it shown below.

Cutting 1: Julian Day 105 (Apr 15)

Cutting 2: Julian Day 133 (May 13)

Cutting 3: Julian Day 161 (Jun 10)

Cutting 4: Julian Day 189 (Jul 8)

Cutting 5: Julian Day 217 (Aug 5)

Cutting 6: Julian Day 245 (Sep 2)

Cutting 7: Julian Day 288 (Oct 15)

Eq. 179

For Pasture and Urban Lawn crop types a slightly different approach is used. For these crops the supposition is that cuttings happen every 7 days to simulate cattle grazing and mowing.

Winter Wheat Specific Algorithms

For annuals crops Winter Wheat is the only crop that is treated differently than the others. The reason is because Wheat is a crop that starts growing during the winter season and ends by the beginning of the summer. Following this assumption the model only starts counting heat units for Winter Wheat on June 1 of each year and keeps accumulating them until May 31 of the next year. So Winter Wheat can start growing on December 15 as it is set up by default. On May 31 all variables related to Winter Wheat are set up back to zero in order to start another year. Consequently for each run during the 1st year there is not a Winter Wheat crop growing until the 2nd year.

REFERENCES

- Ainsworth, E.A. and S.P. Long. 2005. What we have learned from 15 years of free-air CO₂ enrichment (FACE)? A meta-analytic review of the responses of photosynthesis, canopy properties and plant production to rising CO₂. *New Phytologist*, 154(351-372).
- Allen, R.G., L.S. Pereira, D. Raes, M. Smith. 1998. Crop Evapotranspiration – Guidelines for computing crop water requirements – FAO Irrigation and Drainage Paper 56. Rome, Italy.
- Borg, H. and D.V. Grimes. 1986. Depth development of roots with time: An empirical description. *Trans. ASAE* 29:194-197.
- Doorenbos, J. and A.H. Kassam. 1979. Yield response to water. Irrigation and Drainage Paper 33. Food Agric. Org. United Nations, Rome.
- Hatfield, J.L. 2008. The effects of climate change on agriculture, land resources, water resources and biodiversity. U.S. Climate Change Science Program: Synthesis and Assessment Product 4.3.
- Huntington, T.G. 2004. Climate change, growing season length, and transpiration: Plant response could alter hydrologic regime. *Plant Biology*, 6(651-653).
- Jensen, M.E., R.D. Burman, and R.G. Allen (ed). 1990. Evapotranspiration and irrigation water requirements. ASCE Manuals and Reports on Engineering Practice No. 70. ASCE, N.Y. 332 pp.
- Jones, C.A. 1985. C-4 Grasses and Cereals. John Wiley & Sons, Inc., New York.
- Kimball, B.A., K. Kobayashi, and M. Bindi. 2002. Responses of agricultural crops to Free-Air CO₂ Enrichment. *Advances in Agronomy*, 77(293-368).
- Kimball, B.A. 2010. Lessons from FACE: CO₂ effects and interactions with water, nitrogen and temperature. Chapter 5 in “Handbook of Climate Change and Agroecosystems.” Daniel Hillel and Cynthia Rosenzweig eds.
- Long, S.P., E.A. Ainsworth, A.D.B. Leakey, J. Nosberger, and D.R. Ort. 2006. Food for thought: Lower-than-expected crop yield stimulation with rising CO₂ concentrations. *Science*, 312(1918 – 1921).
- Monteith, J.L. 1965. Evaporation and environment. *Symp. Soc. Exp. Biol.* 19:205-234.
- Monteith, J.L. 1973. Principles of Environmental Physics. Edward Arnold, London.

- Monteith, J.L. 1977. Climate and the efficiency of crop production in Britain. *Phil. Trans. Res. Soc. London Ser. B.* 281:277-329.
- Neitsch, S.L., J.G. Arnold, J.R. Kiniry, and J.R. Williams. (2005) Soil and Water Assessment Tool Theoretical Documentation: Version 2005. <http://www.brc.tamus.edu/swat/doc.html>
- Penman, H.L. 1956. Evaporation: An introductory survey. *Netherlands Journal of Agriculture Science* 4:7-29.
- Stockle, C.O., J.R. Williams, N.J. Rosenberg, and C.A. Jones. 1992. A method for estimating the direct and climatic effects of rising atmospheric carbon dioxide on growth and yield of crops: Part I--Modification of the EPIC model for climate change analysis. *Agric. Systems* 38:225-238.
- Thornthwaite, C. W. 1948. An approach toward a rational classification of climate. *Geographical Review* 38:55-94
- Uchijima, Z., T. Udagawa, T. Horie, and K. Kobayashi. 1968. The penetration of direct solar radiation into corn canopy and the intensity of direct radiation on the foliage surface. *J. Agron. Meteorol. Tokyo* 3:141-151.
- Williams, J.W., R.C. Izaurralde, and E.M. Steglich. 2008. Agricultural Policy/Environmental Extender Model: Theoretical Documentation, Version 0604. Blackland Research and Extension Center, Temple, Texas.